MAT 171: Chapter 1

August 21, 2020

Recitations for chapter 1
Review

Arizona State University

Factoring Polynomials (1)

- Factor

$$10x^2 + 11x - 6$$

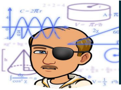
Hint: See how we can get 11 from $-6 \cdot 10$.

$$10x^2 + 11x - 6 \rightarrow \left(10x^2 + 15x\right) + (-4x - 6)$$

$$\xrightarrow{\text{get terms in paranthesis same}} 5x(2x+3) - 2(2x+3)$$

$$(5x-2) \cdot (2x+3)$$

$$x = \frac{2}{5}, x = -\frac{3}{2}$$



Factoring Polynomials (2)

- Factor
$$6x^3 + 9x^2 - 60x$$

$$\begin{aligned} 6x^3 + 9x^2 - 60x &= 3x\left(2x^2 + 3x - 20\right) \\ &= 3x\left(2x^2 \underbrace{-5x + 8x}_{=3x} -20\right) \\ &= 3x\big(x(2x-5) + 4(2x-5)\big) \\ &= 3x\big((x+4)\cdot(2x-5)\big) \end{aligned}$$

- So, our solutions are $x = 0$, $x = \frac{5}{2}$, and $x = -4$.

Factoring Polynomials (3)

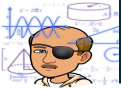
- Factor

$$x^3 + 4x^2 - 9x - 36$$

$$\begin{aligned} x^3 + 4x^2 - 9x - 36 &= x^2(x+4) - 9(x+4) \\ &= (x^2+9) \cdot (x+4) \\ &= (x+3) \cdot (x-3) \cdot (x+4) \end{aligned}$$

- So, our solutions are $x = 3$, $x = -3$, and $x = -4$.

- Notice, here we factored the terms separately. This process is all about writing the polynomial in an "easier" form.



Simplification (1)

- Simplify
$$\frac{x^2-2x-8}{x^2-9} \div \frac{x-4}{x+3}$$
- Notice, $x^2-9=(x+3)\cdot(x-3)$ and $x^2-2x-8=(x-4)\cdot(x+2)$, so we have
$$\frac{(x-4)\cdot(x+2)}{(x+3)\cdot(x-3)}\frac{x+3}{x-4}=\frac{x+2}{x-3}$$

Simplifying (2)

- Simplify

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

The trick here is to match the denominators:

$$\frac{1}{h} \cdot \left(\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)} \right)$$
$$= \frac{1}{h} \cdot \left(\frac{-h}{x(x+h)} \right)$$
$$= \boxed{\frac{-1}{x(x+h)}}$$

Simplifying (3)

- Simplify

$$\frac{\frac{3}{x-2} - \frac{4}{x+2}}{\frac{7}{x^2-4}}$$

Split the numerator and denominator to make things clearer, noting dividing in denom is same as multiplying in numerator and $(x-2)\cdot(x+2) = x^2-4$:

$$\frac{3(x+2) - 4(x-2)}{x^2-4} \cdot \frac{x^2-4}{7}$$
$$= \frac{-x+14}{7}$$

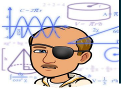
Simplifying (4)

- Simplify

$$\frac{x-5}{\sqrt{x+11}-4}$$

The trick here is to rationalize the denominator, i.e. multiply by the denominator but flip the - to a $+$, so as to cancel terms when squaring!

$$\begin{aligned}\frac{x-5}{\sqrt{x+11}-4}\cdot\left(\frac{\sqrt{x+11}+4}{\sqrt{x+11}+4}\right) \\ &= \frac{(x-5)\cdot(\sqrt{x+11}+4)}{x+11-16} \\ &= \sqrt{x+11}+4\end{aligned}$$



True or False

- Is the following equality true?

$$\sqrt{x^2 + y^2} = x + y$$

- No. For example, let $x = 2, y = 3$. Then $\sqrt{x^2 + y^2} = \sqrt{13}$, but $x + y = 5$, so they are not equal. Additionally, note after squaring both sides, we have

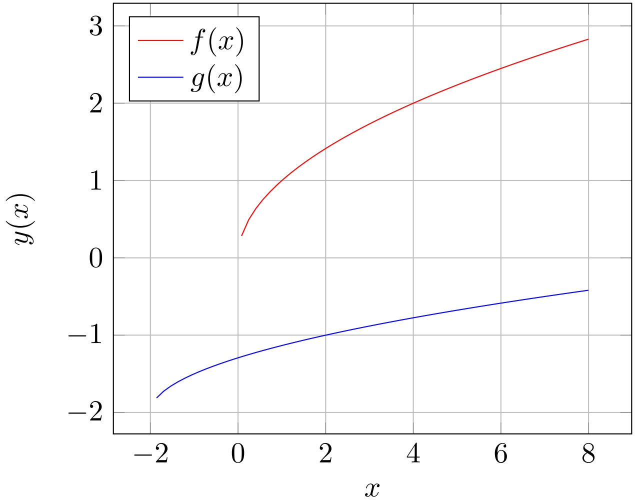
$$(x + y)^2 = x^2 + y^2 + 2xy \neq x^2 + y^2$$

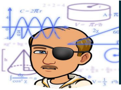
so the two are not equal.

Week 1:
Question 1

- Graph the function $f(x) = \sqrt{x}$. Then graph the function $g(x) = \frac{1}{2}\sqrt{x+2} - 2$ using transformations. List the transformations and give the domain and range of $g(x)$. Make sure you label your graph.
- See the figure (**??**) on next page: The domain for $f(x)$ is $(0, \infty)$, the range is $[0, \infty)$, the domain of $g(x)$ is $(-2, \infty)$ and the range is $[-2, \infty)$.

Week 1:
Question 1





Week 1:
Question 2

- Find the function $g(r) = \frac{4r^2-1}{r^2}$ when $r = -x$. Explain why this shows that $g(r)$ is an even function.
- Here we plug in $-x$ where ever we see r. The tricky part is to keep an eye on the negative sign. For this reason, its suggested to write $r = (-x)$. Lets plug in and see what we get:

$$\begin{aligned} g(-x) &= \frac{4(-x)^2 - 1}{(-x)^2} \\ &= \frac{4x^2 - 1}{x^2} \end{aligned}$$

This is an even function because $h(x) = h(-r)$ (just calling x r here.)

Week 1:
Question 3

- Find and simplify the difference quotient of the function $f(x) = -x^2 + 2x - 1$. The difference quotient is

$$\frac{f(x+h) - f(x)}{h}$$

- We know $f(x)$, but we need to calculate $f(x+h)$. Let's do it.

$$\begin{aligned} f(x+h) &= -(x+h)^2 + 2(x+h) - 1 \\ &= -x^2 - h^2 - 2xh + 2x + 2h - 1 \end{aligned}$$

Week 1:
Question 3

Then, we have that the difference quotient is

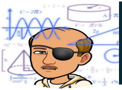
$$\frac{f(x+h)-f(x)}{h} = \frac{-x^2-h^2-2xh+2x+2h-1-\left(-x^2+2x-1\right)}{h}$$

$$\xrightarrow{\text{distribute the negative sign, and combine terms of same order}}$$

$$= \frac{\cancel{-x^2+x^2}-h^2+-2hx+2h\cancel{+2x-2x}\cancel{-1+1}}{h}$$

$$= \frac{-h^{\cancel{2}}-2\cancel{h}x+2\cancel{h}}{\cancel{h}}$$

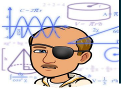
$$= \boxed{-h-2x+2}$$



Week 1:
Question 4: part a

- The function $h(t) = -4t^2 + 8t + 32$ models the height, h, of a thrown ball after t seconds.

- We find the y-intercept by setting time, t, equal to 0. Therefore the intercept is 32, which we interpret as the height you throw the ball from.



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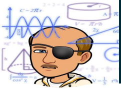

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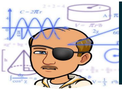
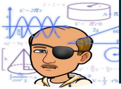


Week 1:
Question 4: part b

- Find the interval of time where the ball's height is decreasing.
- There are multiple ways to do this. For one, we can plot and see where the maximum of the function is and find the time where this occurs. See figure(1).
- Using calculus, you can find the derivative of $h(t)$ with respect to time, and then see when that is negative.

$$h'(t) = \frac{\mathrm{d}h}{\mathrm{d}t} = -8t + 8$$

which is negative when $t > 1$. Therefore, after 1 second the height decreases.
However, looking at the plot, 1, is probably more in line with what we've just learned. There, we can see the interval the ball is decreasing is t in $[1, 4]$.



Week 1:
Question 4: part b

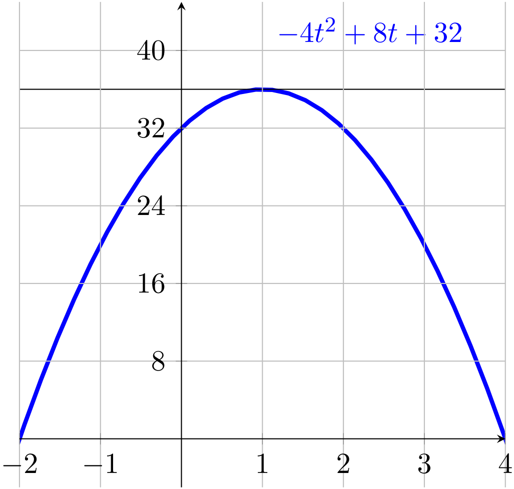


Figure: Figure 2 week 1

Week 1:
Question 4c

- What is the domain and range of this function in the context of the problem?
- The domain we can think of as the t values, the time the ball is thrown. We start at 0. Similarly, we do not have negative distance, the ground is 0. Therefore, the maximum number of time is the time at which the ball reaches the ground which is at $t = 4$. We can find this by factoring the equation:

$$\begin{aligned} -4t^2 + 8t + 32 &= -4\Big(t^2 - 2t - 8\Big) = 0 \\ &= -4\Big((t^2 - 4t) + (2t - 8)\Big) = 0 \\ &= -4(t+2)(t-4) = 0 \end{aligned}$$

- So our two solutions are at $t = -2$ and $t = 4$, but our domain tells us $t > 0$. So the maximum time is $t = 4$,